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PARAMETRIC STUDY OF OPTIMUM FIBER ORIENTATION FOR FILAMENTARY SHEET

Prepared by

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ANALYSIS METHODS - OPTIMIZATION

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FOREWORD

This work was conducted by Mr. R. S. Sandhu, Exploratory Development Group, Advanced Composites Branch, at the Air Force Flight Dynamics Laboratory, under project 4364, "Filamentary Composite Structures," Task 436402, "Design Allowable and Criteria."

The manuscript was released by the author in August 1971. This Technical Memorandum has been reviewed and is approved.

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ABSTRACT

This report contains the results of the Study of the influence of stress states and strength parameters on the optimum orientation. Using Hills' strength theory for anisotropic materials, a criterion is established to determine the optimum fiber orientation.

I. Introduction

At present, most failure criteria proposed for anisotropic materials are generalizations of those originally formulated for isotropic materials. Of all the hypotheses proposed for anisotropic materials, two failure criteria have been most widely used for composites. They are the maximum strain theory and Hill's criterion modified by Tsai [1]. Tsai's strength theory can be used, for a given set of strength characteristics of a planar orthotropic material and a given orientation of material axes, to determine a stress state at failure. On the same basis, it is reasonable to expect that for a given set of strength parameters and a given stress state there exists an orientation of the material axes which maximizes the strength. An investigation of this premise led Sandhu [2] and Brandmaier [3] to conclude that optimum orientations are determinable. When the shear strength S is less than the transverse strength Y, the material attains the maximum strength with the major material axis coinciding with the major principal stress direction. However, in case the shear strength is greater than the transverse strength, the fiber orientation maximizing the strength may be different from the principal stress direction. Determination of the optimum orientation then becomes more involved. In this note the problem of the influence of stress states and strength parameters on the optimum orientations will be examined. In the derivation of the conditions which must be satisfied to obtain the optimum orientations, a principal stress state will be assumed.

II. Analysis

In Figure 1, σ_1 and σ_2 are the major and the minor principal stresses respectively acting at a point. The x-axis and y-axis are the major and minor axes of the material symmetry. The angle between the x-axis and the σ_1-axis is θ. σ_{xx}, σ_{yy}, τ_{xy} are the normal and the shear components of the stress at a point in the x-y coordinate system. σ_{xx}, σ_{yy}, τ_{xy} expressed in terms of σ_1, σ_2, and θ are

$$\begin{bmatrix} \sigma_{xx}/\sigma_1 \\ \sigma_{yy}/\sigma_1 \\ \tau_{xy}/\sigma_1 \end{bmatrix} = \begin{bmatrix} \cos^2\theta & \sin^2\theta & 2\sin\theta\cos\theta \\ \sin^2\theta & \cos^2\theta & -2\sin\theta\cos\theta \\ -\sin\theta\cos\theta & \sin\theta\cos\theta & \cos^2\theta-\sin^2\theta \end{bmatrix} \begin{bmatrix} 1 \\ n \\ 0 \end{bmatrix} \tag{1}$$

where $$n = \sigma_2/\sigma_1 \tag{2}$$

and $$\left|\sigma_2/\sigma_1\right| \leqslant 1 . \tag{3}$$

If X and Y are the normal failure stresses such that $X/Y \geqslant 1$ and S is the failure shear stress of the material, Tsai's [1] strength criterion is given by the Equation (4).

$$\left[\frac{\sigma_{XX}}{\sigma_1}\right]^2 - \frac{\sigma_{XX}\sigma_{YY}}{\sigma_1^2} + \left[\frac{X}{Y}\right]^2\left[\frac{\sigma_{YY}}{\sigma_1}\right]^2 + \left[\frac{X}{S}\right]^2\left[\frac{\tau_{XY}}{\sigma_1}\right]^2 = \left[\frac{X}{\sigma_1}\right]^2 \tag{4}$$

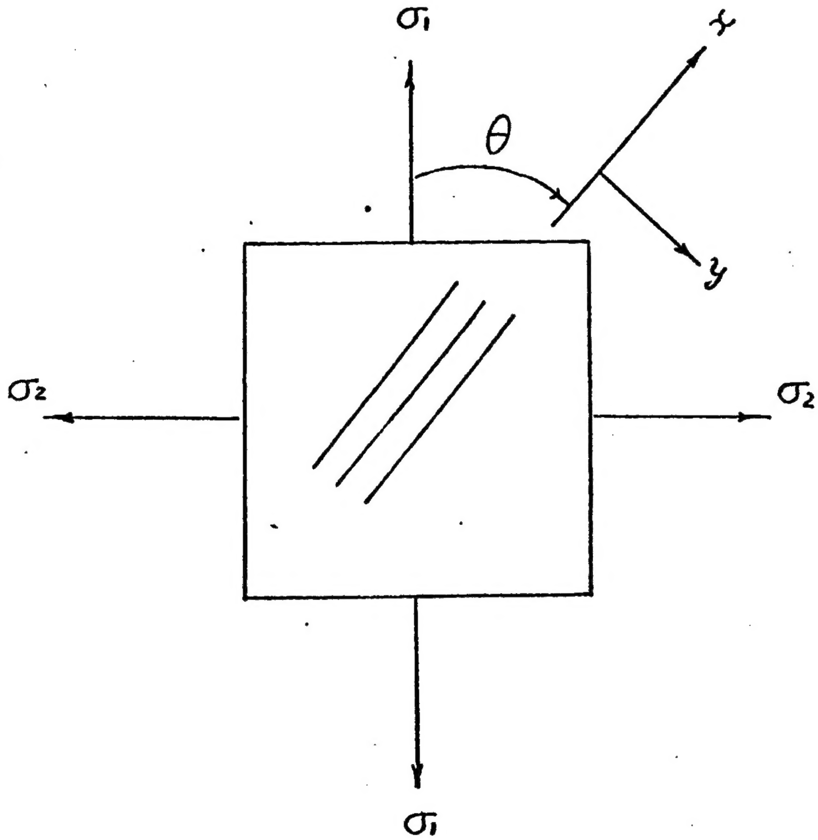

Figure 1. Applied Stresses

Substitution of the Equation (1) in the Equation (4) yields

$$R_x^2 = \left[X/\sigma_1\right]^2 = \left[n^2 - n + \alpha^2\right] \sin^4\theta + \left[-n^2 + 2n-1+2n\alpha^2+(n-1)^2\beta^2\right] \sin^2\theta \cos^2\theta + \left[1-n+\alpha^2 n^2\right] \cos^4\theta \tag{5}$$

or

$$Rx = f(n,\alpha,\beta,\theta) \tag{6}$$

where

$$\alpha = X/Y$$

$$\beta = X/S \tag{7}$$

$$Rx = X/\sigma_1$$

For a given set of α, β, and n, the strength factor, Rx, assumes extrema values when

$$df/d\theta = 0 \tag{8}$$

or

$$2(n-1)\sin\theta\cos\theta \left[\left[(3-\beta^2)\,n-(1+2\alpha^2-\beta^2)\right] \sin^2\theta - \left[(1+2\alpha^2-\beta^2)\,n-(3-\beta^2)\right] \cos^2\theta\right] = 0 \tag{9}$$

The Equation (9) is satisfied for

(a) $n-1 = 0$ (10)

(b) $\sin\theta = 0$ (11)

(c) $\cos\theta = 0$ (12)

(d) $\tan\theta = \pm\sqrt{U/L}$ (13)

where

$$U = n(1+2\alpha^2-\beta^2)-(3-\beta^2) \tag{14}$$

$$L = n(3-\beta^2)-(1+2\alpha^2-\beta^2) \tag{15}$$

The positive sign in the Equation (13) indicates that the angle θ is clockwise from σ_1-axis and the negative sign yields anticlockwise angle θ. Without loss of generality the negative sign will be dropped in the subsequent discussion.

For $n = 1$, the Equation (5) yields

$$\sigma_1/X = Y/X \tag{16}$$

i.e., the maximum stress σ_1 depends upon the transverse strength Y only.

When $\theta = 0^o$, the Equation (5) reduces to

$$\left[\sigma_1/X\right]_{\theta=0^\circ} = \frac{1}{\sqrt{1 - n + \alpha^2 n^2}} \tag{17}$$

Similarly for $\theta = 90^o$, the Equation (5) yields

$$\left[\sigma_1/X\right]_{\theta=90^\circ} = \frac{}{\sqrt{n^2 - n + \alpha^2}} \tag{18}$$

For all stress states defined by n, and for $\alpha > 1$, $\left[\sigma_1/X\right]_{\theta=0^\circ}$ is greater than $\left[\sigma_1/X\right]_{\theta=90^\circ}$; therefore, $\theta = 90^o$ will not be considered.

When $\theta \neq 0^o$, $\theta \neq 90^o$, and $n \neq 1$, the Equation (13) indicates the existence of an orientation θ which may yeild $\left[\sigma_1/X\right]_\theta$ greater than $\left[\sigma_1/X\right]_{\theta=0^\circ}$. The Equation (13) gives rise to real values of θ only when U and L are of the same sign. This depends upon the state of stress and the values of α and β. An examination of the Equations (14) and (15) indicates that real values of θ are obtainable from the Equation (13) for the conditions

$$\text{(i)} \quad -1 \leqslant n \leqslant \frac{3 - \beta^2}{1 + 2\alpha^2 - \beta^2} \quad \text{and} \quad (a^2 + 2) \geqslant \beta^2 \geqslant 1 \tag{19}$$

with the corresponding $U/L < 1$;

$$\text{(ii)} \quad -1 \leqslant n \leqslant \frac{1+2\alpha^2-\beta^2}{3-\beta^2} \quad \text{and } \beta^2 > (a^2+2) \tag{20}$$

with $U/L > 1$.

The next question to be examined is how does $[\sigma_1/X]_\theta$ computed from the Equation (5) for the conditions (19) and (20) compare with $[\sigma_1/X]_{\theta=0^\circ}$. To do this, the Equation (5) is rewritten as

$$[X/\sigma_1]^2_\theta = (1-n+\alpha^2n^2) + (n-1)\sin^2\theta\left[(n-1)(2+\alpha^2-\beta^2)\ U/U+L - U\right] \tag{21}$$

Now $[\sigma_1/X]_\theta \geqslant [\sigma_1/X]_\theta = 0^\circ$ if

$$(n-1)\sin^2\theta\left[(n-1)(2+\alpha^2-\beta^2)\ U/U+L - U\right] \leqslant 0 \tag{22}$$

The condition (22) simplifies to

$$U\left[1/2-1\right] \geqslant 0 \tag{23}$$

The condition (23) is satisfied only for $U < 0$, ie,

$$n \leqslant \frac{3-\beta^2}{1+2\alpha^2-\beta^2} \tag{24}$$

Hence the Equation (13) subject to the condition (19) will yield a value of θ which will maximize $[\sigma_1/X]_\theta$.

III . Conclusions

The results of this study indicate that when $(\alpha^2 + 2) \geqslant \beta^2$, the maximum strength of the composite is obtained by aligning the major material axis at an angle θ $(0^o \leqslant \theta \leqslant 45^o)$ computed from Equation (13) for $-1 \leqslant n \leqslant \frac{3-\beta^2}{1+2\alpha^2-\beta^2}$. If the conditions given in Equation (19) are not satisfied, maximum strength will result by aligning the major material axis along the major principal stress direction. For example for the graphite/epoxy composite [3], X = 125 Ksi; Y = 5 Ksi, and S = 10 Ksi, the condition $(\alpha^2 + 2) > \beta^2$ is satisfied and, therefore, maximum strength can be obtained for $-1.0 \leqslant n < -0.14$ by aligning the fibers at an angle θ obtained from Equation (13). For $1.0 \geqslant n \geqslant -0.14$, the orientation angle θ must coincide with the major principal stress direction in order to maximize the strength. Similarly for Courtaulds HTS filament with 4617 resin, X = 244 Ksi, Y = 17.9 Ksi, and S = 18.5 Ksi, the condition $(\alpha^2 + 2) > \beta^2$ is again satisfied. The maximum strength is obtained by aligning the fibers with the σ_1 direction when $-0.86 \leqslant n \leqslant 1$ and with θ obtained from the Equation (13) when $-1 \leqslant n \leqslant -0.86$.

The conclusions drawn herein depend upon Tsai's strength criterion being valid for the material under study. If, as a result of current research efforts, a new criterion is evolved, results similar to those obtained in this study can be generated.

IV. References

1. V. D. Azzi and S. W. Tsai, "Anisotropic Strength of Composites," Experimental Mechanics (SESA), September 1965.

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3. H. E. Brandmaier, "Optimum Filament Orientation Criteria," Journal of Composite Materials, Vol 4, July 1970.